9:00 - 11:00
32-155

MIT ID# (last four digits) SOLUTIONS

Unified Quiz FTM2

October 17, 2007

M - PORTION

EXAM SCORING:

#1M and FINAL SCORE	

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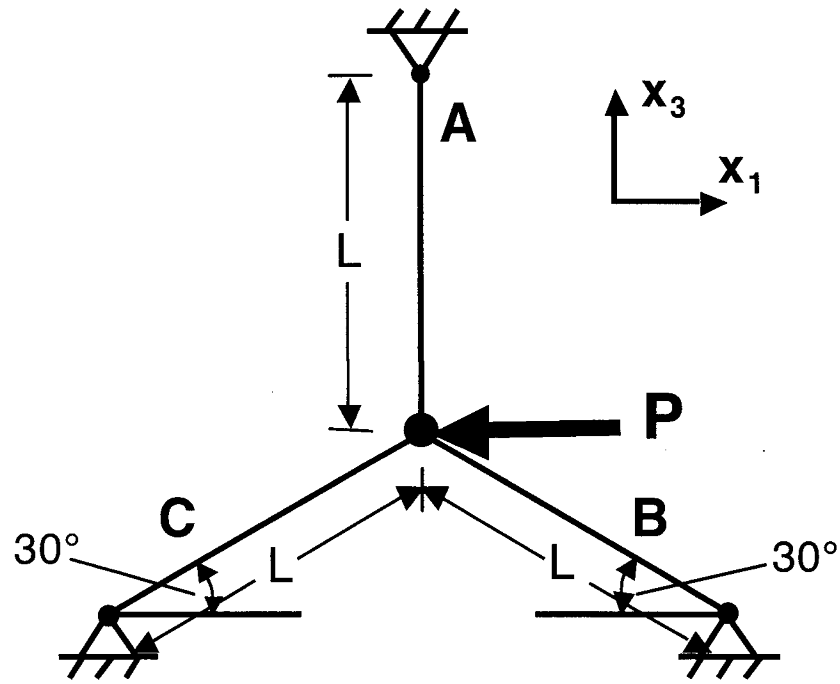
(last four digits)

PROBLEM #1M

A three-bar truss is pinned at the juncture of the three bars. Each bar is of length L, cross-sectional area A, and made of a material with modulus E. The other end of each bar is pinned allowing rotation of the bar at that point. Prior to being placed under load, bar A is aligned vertically, while bars B and C make 30° angles with the horizontal direction, as shown. A horizontal load of magnitude P is applied in the negative x_1 direction.



(a) What is the "class/category" of this structural configuration (Dynamic, Statically Determinate, Statically Indeterminate)? **Clearly** explain your reasoning.

Draw the Free Body Diagram for each support:

H →
V ↑

There are 3 of these giving a total of 6 reactions.

In a 2-D system, there are 3 degrees of freedom

\# of reactions (6) > # d.o.f. (3)

⇒ cannot solve just via equilibrium

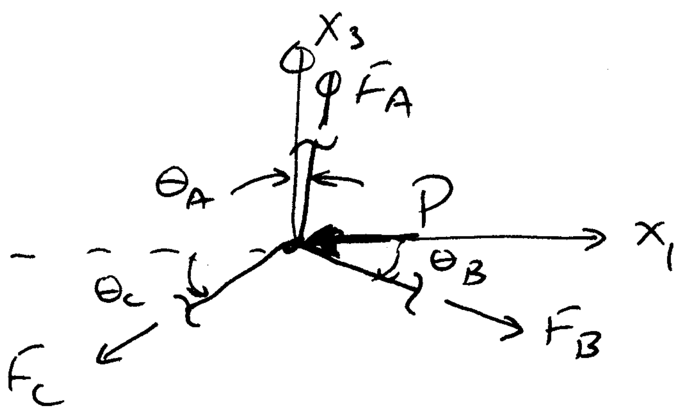
⇒ $\boxed{\text{Statically Indeterminate}}$

PROBLEM #1M (continued)

(b) Set up the equations available to determine the bar loads. **Clearly explain** the approach needed and the steps taken. Indicate whether there are sufficient equations to determine the loads or indicate the additional information needed for such. **Do not solve the equations.**

→ Apply the 3 "Great Principles": - Equilibrium
- Compatibility
- Constitutive Relations

→ Consider a subsystem around the joint with the origin of the axis system at the joint



Each of the bars can rotate, so define angles (relative to x_1 and x_3) that these end up making. These have magnitudes with directions as defined:

θ_A, θ_B, θ_c

The bar forces can be resolved into x_1 and x_3 components using these angles.

Do this and apply...

→ <u>Equilibrium</u>

$$\sum F_{x_1} = 0 \quad \xrightarrow{+} \quad \Rightarrow -P + F_A \sin\theta_A + F_B \cos\theta_B - F_c \cos\theta_c = 0 \quad (1)$$

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PROBLEM #1M (continued)

$\sum F_{x_3} = 0 \quad \uparrow + \quad \Rightarrow F_A \cos\theta_A - F_B \sin\theta_B - F_C \sin\theta_C = 0$ (2)

$\sum M = 0$ ... no information since all bar forces act through the joint

Now move on to use....

→ Constitutive Relations

For a bar the general constitutive relation is:

$$F = \frac{AE}{L}\delta$$

Here we have A (area), E (modulus) and L (length) the same for each bar.

Define the change in length of a bar as δ using the bar letter as a subscript with $+ \Rightarrow$ increase in length

so:

$$F_A = AE/L \; \delta_A \qquad (3)$$
$$F_B = AE/L \; \delta_B \qquad (4)$$
$$F_C = AE/L \; \delta_C \qquad (5)$$

* Now take stock in the number of unknowns:

bar forces: F_A, F_B, F_C

bar angles: $\theta_A, \theta_B, \theta_C$

bar displacements: $\delta_A, \delta_B, \delta_C$

$\Rightarrow$ 9 unknowns with 5 equations

so now go to....

→ Compatibility

to try to find 4 equations

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PROBLEM #1M (continued)

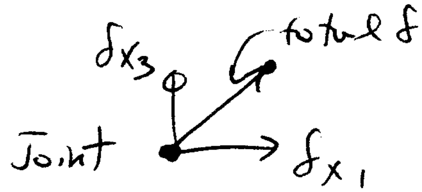
Define the displacement of the joint of the bars as two components: δx_1 and δx_3 with positive (+) in positive direction of axes.

Now relate ~~each~~ bar elongation and bar angle to these two items (2 new unknowns for 11 total)

If one can get 6 equations via this technique, there will be 11 equations in 11 unknowns and thus

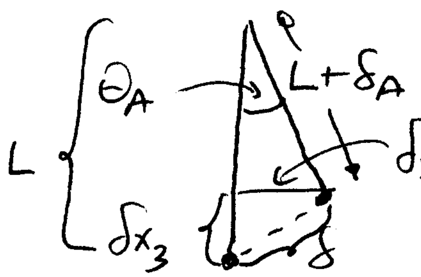
$$\Rightarrow \boxed{\text{SOLVABLE}}$$

→ Proceed:



$$\delta = \sqrt{(\delta x_1)^2 + (\delta x_3)^2}$$

→ Bar A



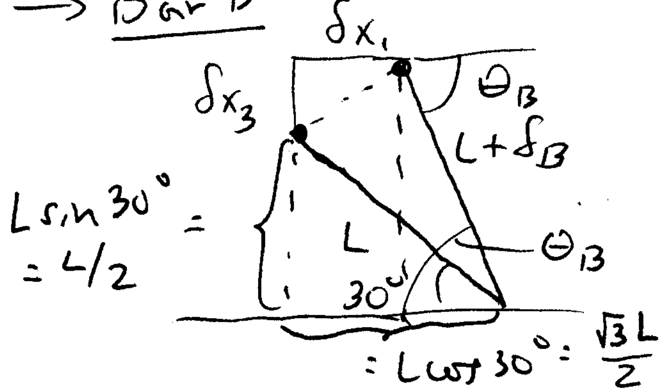
$$(L+\delta_A)\sin\theta_A = \delta x_1$$

$$\Rightarrow \theta_A = \sin^{-1}\frac{\delta x_1}{(L+\delta_A)} \quad (6)$$

via the right triangle:

$$(L-\delta x_3)^2 + \delta x_1^2 = (L+\delta_A)^2 \quad (7)$$

→ Bar B



via the right triangle:

$$\left(\frac{\sqrt{3}}{2}L - \delta x_1\right)^2 + \left(\frac{L}{2} + \delta x_3\right)^2 = (L+\delta_B)^2 \quad (8)$$

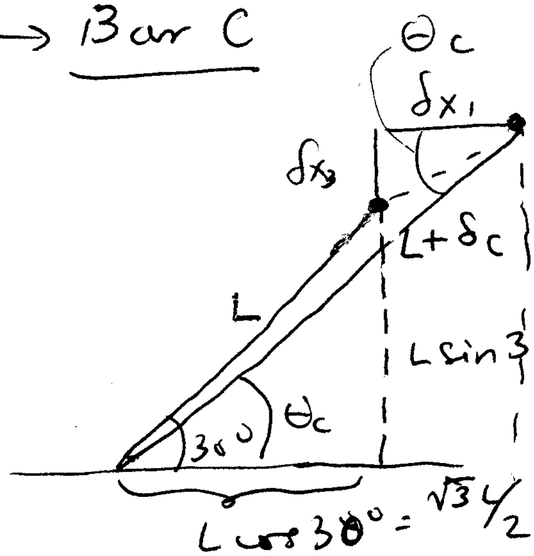
and:

$$(L+\delta_B)\sin\theta_B = (L/2 + \delta x_3)$$

$$\Rightarrow \theta_B = \sin^{-1}\left[\frac{L/2+\delta x_3}{L+\delta_B}\right] \quad (9)$$

PROBLEM #1M (continued)

→ Bar C



via the right triangle:

$$\left(\frac{\sqrt{3}}{2}L+\delta x_1\right)^2+\left(L/2+\delta x_3\right)^2 = \left(L+\delta_c\right)^2 \quad (10)$$

and:

$$(L+\delta_c)\sin\theta_c = \left(\frac{L}{2}+\delta x_3\right)$$

$$\Rightarrow \theta_c = \sin^{-1}\left[\frac{\frac{L}{2}+\delta x_3}{L+\delta_c}\right] \quad (11)$$

11 equations (as numbered) in 11 unknowns:

$f_A, f_B, f_c, \theta_A, \theta_B, \theta_c, \delta_A, \delta_B, \delta_c, \delta x_1, \delta x_3$

$\Rightarrow$ $\boxed{\text{SOLVABLE}}$